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On the Stationarity and Ergodicity of Fading Channel Simulators Based on Rice's Sum-of-Sinusoids

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Channel simulators based on Rice's sum-of-sinusoids are playing an important role in fading channel modelling. However, the parameters of the sum-of-sinusoids have to be determined meticulously to fully exploit the potential that this powerful procedure has to offer. This paper presents general conditions under which the sum-of-sinusoids procedure results in a stationary and ergodic channel simulator. Moreover, with the help of the introduced conditions, several established parameter computations methods will be investigated with respect to their usability to design stationary and ergodic fading channel generators. It turns out that if and only if the gains and frequencies are constant quantities and the phases are random variables, then the sum-of-sinusoids defines a stationary and ergodic process.

KEY WORDS: Deterministic processes; fading channel simulators; mobile channel modelling; stochastic processes; sum-of-sinusoids principle.

1. INTRODUCTION

The sum-of-sinusoids principle was originally introduced in Rice's seminal work [1,2] as a method to model Gaussian noise with given correlation properties. In present days, this principle is very popular in mobile communications, since it enables the design of efficient and flexible mobile fading channel simulators. Its application ranges from the development of fading channel generators for simple time-variant channels [3,4] over frequency-selective channels [5,6] up to elaborated space–time wideband channels [7,8]. However, when Rice's original method is used to compute the model parameters, then the period of the resulting fading process is merely proportional to the number of sinusoids [6]. This is a serious drawback, since it prevents keeping the realization expenditure low. Fortunately, many alternative methods (e.g., [3–5,9–14]) have been developed to avoid this drawback.

Apart from the period, other performance measures are also important. For example, it is of central importance to know the conditions under which a finite sum of harmonic functions with random parameters results in a stationary and ergodic channel simulator. To provide a solution to this problem is the topic of the present paper. Here, general conditions are stated guaranteeing that a fading channel simulator based on the sum-of-sinusoids principle is not only stationary but also ergodic. In practical applications, ergodic channel simulators are beneficial, since they enable to minimize the number of required simulation runs.

The organization of the paper is as follows. Section 2 describes the reference model defined by an infinite number of sinusoids. Limiting the number of sinusoids leads to the simulation model introduced in Section 3. Section 4 introduces performance criteria, which are used in Section 5 to classify the simulation models. Section 6 applies the proposed concept to several parameter computation methods. Finally, Section 7 draws the conclusion.

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2. THE REFERENCE MODEL

For our purpose, it is sufficient to consider Rayleigh fading. A Rayleigh process, $\zeta(t)$, is defined as

$$\zeta(t) = |\mu_1(t) + j\mu_2(t)|, \tag{1}$$

where $\mu_1(t)$ and $\mu_2(t)$ are two zero-mean real Gaussian processes, each with variance σ_0^2. As usual, it is assumed that $\mu_1(t)$ and $\mu_2(t)$ are uncorrelated. According to Clarke's [15] popular two-dimensional isotropic scattering model, the autocorrelation function $r_{\mu_i\mu_i}(\tau)$ of $\mu_i(t)$ $(i = 1, 2)$ is given by

$$r_{\mu_i\mu_i}(\tau) = \sigma_0^2\, J_0(2\pi f_{\max}\tau), \tag{2}$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind and $f_{\max}$ is the maximum Doppler frequency. The principle of Rice's sum-of-sinusoids [1,2] is based on a superposition of an infinite number of weighted sinusoids with equidistant frequencies and random phases. Using this principle, a Gaussian process $\mu_i(t)$ can be modelled as

$$\mu_i(t) = \lim_{N_i\to\infty} \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}), \tag{3}$$

where N_i denotes the number of sinusoids. According to Rice [1,2], the gains $c_{i,n}$ and frequencies $f_{i,n}$ in the expression above are given by

$$c_{i,n} = 2\sqrt{\Delta f_i\, S_{\mu_i\mu_i}(f_{i,n})}, \tag{4}$$

$$f_{i,n} = n\,\Delta f_i, \tag{5}$$

respectively, where $i = 1, 2$, and $n = 1, 2, \ldots, N_i$. In Eq. (3), the phases $\theta_{i,n}$ are assumed to be random variables having a uniform distribution in the interval (0, 2π]. The quantity Δf_i appearing in Eqs. (4) and (5) is chosen in such a way that the relevant one-sided frequency range is completely covered by Eq. (5). The symbol $S_{\mu_i\mu_i}(f)$ in Eq. (4) denotes the Doppler power spectral density, which is related to the autocorrelation function $r_{\mu_i\mu_i}(\tau)$ via the Fourier transform.

Since the number of sinusoids N_i is infinite in Eq. (3), a software or hardware realization of $\mu_i(t)$ does not exist. Nevertheless, $\mu_i(t)$ in Eq. (3) is useful because it describes the reference model. A reference model is important mainly for two reasons. First, the non-realizable stochastic reference model is the starting point for the derivation of a realizable stochastic (or deterministic) simulation model. And second, the reference model enables us to measure the performance of the resulting simulation model described in Section 3.

3. THE SIMULATION MODEL

A realizable stochastic simulation model is obtained by using only a finite number of sinusoids N_i. Hence,

$$\hat{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}), \tag{6}$$

where the gains $c_{i,n}$ and frequencies $f_{i,n}$ are still constants, and the phases $\theta_{i,n}$ are again uniformly distributed random variables.

Now, we consider the phases $\theta_{i,n}$ as outcomes (realizations) of a random generator with a uniform distribution in the interval (0, 2π]. In this case, the phases $\theta_{i,n}$ are real-valued constant quantities, and the stochastic process $\hat{\mu}_i(t)$ results in a sample function denoted by

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}). \tag{7}$$

Note that different realizations of the sets $\{\theta_{i,n}\}$ give different sample functions $\tilde{\mu}_i(t)$. Note also that the stochastic process $\hat{\mu}_i(t)$ can be interpreted as a family (or an ensemble) of sample functions, i.e.,

$$\hat{\mu}_i(t) = \{\tilde{\mu}_i(t) \mid t \in \mathbb{R}\}. \tag{8}$$

Since the sample function $\tilde{\mu}_i(t)$ is completely deterministic, we call $\tilde{\mu}_i(t)$ a deterministic process. Such a process can easily be implemented on a hardware or software platform. The realization of a deterministic process $\tilde{\mu}_i(t)$ in form of a hardware system or a software program is called a *deterministic simulation model*.

The relationships between reference models, stochastic simulation models, and deterministic simulation models are shown in Fig. 1. This figure illustrates the following interpretations:

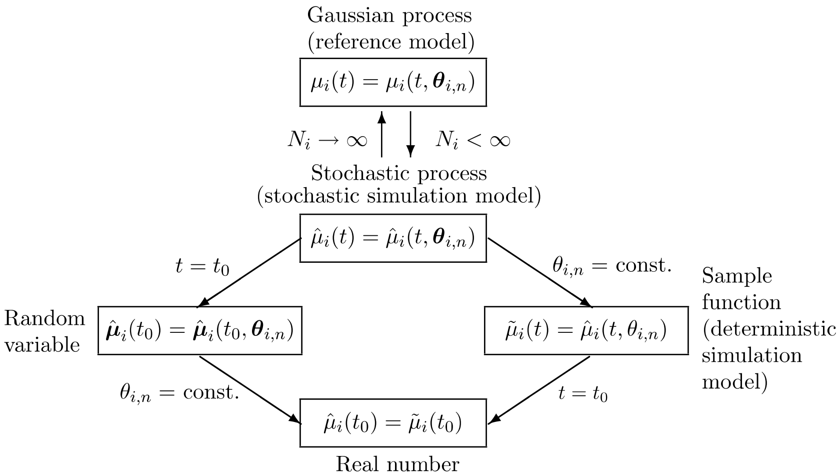

Fig. 1. Relationships between a Gaussian process (reference model), stochastic process (stochastic simulation model), sample function (deterministic simulation model), random variable, and a real number.

1. If $\theta_{i,n}$ is a random variable and $N_i \to \infty$, then $\hat{\mu}_i(t)$ tends to the Gaussian process $\mu_i(t)$ describing the reference model.
2. If $\theta_{i,n}$ is a random variable and N_i is finite, then $\hat{\mu}_i(t)$ is a stochastic process, i.e., a family (or an ensemble) of sample functions $\tilde{\mu}_i(t)$. The realization of the stochastic process $\hat{\mu}_i(t)$ is called the stochastic simulation model.
3. If $\theta_{i,n}$ is fixed, then $\hat{\mu}_i(t)$ results in a specific sample function $\tilde{\mu}_i(t) = \hat{\mu}_i(t, \theta_{i,n})$ called a deterministic process. Its realization is said to be a deterministic simulation model.
4. If $t = t_0$ is fixed and $\theta_{i,n}$ is a random variable, then $\hat{\mu}_i(t_0) = \{\tilde{\mu}_i(t) \mid t = t_0\}$ is a random variable.
5. If both $t = t_0$ and $\theta_{i,n}$ are fixed, then $\hat{\mu}_i(t_0)$ is a real number equalling $\tilde{\mu}_i(t_0)$.

For the performance evaluation of the stochastic simulation model, it is important to know the conditions for which the stochastic process $\hat{\mu}_i(t)$ is stationary and ergodic. A clear understanding of stationary and ergodic processes is important for the intention of the paper. We will therefore review these two terms briefly.

4. PERFORMANCE CRITERIA

4.1. Stationarity

A stochastic process $\hat{\mu}_i(t)$ is said to be *first-order stationary* (FOS) [16, p. 392] if the first-order density of $\hat{\mu}_i(t)$ is independent of time t, i.e.,

$$\hat{p}_{\mu_i}(x; t) = \hat{p}_{\mu_i}(x; t + c) \equiv \hat{p}_{\mu_i}(x) \tag{9}$$

holds for all values of t and $c \in \mathbb{R}$. This implies that the mean and the variance of $\hat{\mu}_i(t)$ are independent of time as well.

A stochastic process $\hat{\mu}_i(t)$ is called *wide-sense stationary* (WSS) [16, p. 388] if $\hat{\mu}_i(t)$ satisfies the following two conditions:

(1) The mean of $\hat{\mu}_i(t)$ is constant, i.e.,

$$E\{\hat{\mu}_i(t)\} = \hat{m}_{\mu_i} = \text{const.} \tag{10}$$

(2) The autocorrelation function of $\hat{\mu}_i(t)$ depends only on the time difference $\tau = t_1 - t_2$, i.e.,

$$\hat{r}_{\mu_i\mu_i}(t_1, t_2) = \hat{r}_{\mu_i\mu_i}(\tau) = E\{\hat{\mu}_i(t)\hat{\mu}_i(t + \tau)\} \tag{11}$$

for all t_1 and t_2.

4.2. Ergodicity

A stochastic process $\hat{\mu}_i(t)$ is *mean-ergodic* if its ensemble average $\hat{m}_{\mu_i}$ equals the time average $\tilde{m}_{\mu_i}$ of $\tilde{\mu}_i(t)$, i.e.,

$$\hat{m}_{\mu_i} = \tilde{m}_{\mu_i} := \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \tilde{\mu}_i(t) \, dt. \tag{12}$$

The stochastic process $\hat{\mu}_i(t)$ is *autocorrelation-ergodic* if its autocorrelation function $\hat{r}_{\mu_i\mu_i}(\tau)$ equals the time autocorrelation function $\tilde{r}_{\mu_i\mu_i}(\tau)$ of $\tilde{\mu}_i(t)$: i.e.,

$$\hat{r}_{\mu_i\mu_i}(\tau) = \tilde{r}_{\mu_i\mu_i}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \tilde{\mu}_i(t)\tilde{\mu}_i(t+\tau)\ \mathrm{d}t. \tag{13}$$

Remember that an ergodic process is always stationary, but a stationary process needs not to be ergodic [16].

5. CLASSIFICATION OF CHANNEL SIMULATORS

In the following, it is assumed that $c_{i,n} \neq 0$ and $f_{i,n} \neq 0$ for all $n = 1, 2, \ldots, N_i$ and $i = 1, 2$. Furthermore, we impose on the sum-of-sinusoids model that the absolute values of all frequencies, $|f_{i,n}|$, are different, i.e., (i) $|f_{i,1}| \neq |f_{i,2}| \neq \cdots \neq |f_{i,N_i}|$ for $i = 1, 2$ and (ii) $\{|f_{1,n}|\}_{n=1}^{N_1} \cap \{|f_{2,n}|\}_{n=1}^{N_2} = \emptyset$, where $\emptyset$ denotes the empty set. The former condition (i) is introduced as a measure to avoid intra-correlations, i.e., correlations within $\tilde{\mu}_i(t)$ ($i = 1, 2$), and the latter condition (ii) ensures that the cross-correlation (inter-correlation) of $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ is zero.

5.1. Class I Channel Simulators

The channel simulators of Class I are defined by the set of stochastic processes $\hat{\mu}_i(t)$ [see (6)] with constant gains $c_{i,n}$, constant frequencies $f_{i,n}$, and random phases $\theta_{i,n}$, which are uniformly distributed in the interval (0, 2π]. In this case, the first-order density of $\hat{\mu}_i(t)$ is given by [17]

$$\hat{p}_{\mu_i}(x) = 2\int_0^\infty \left[\prod_{n=0}^{N_i} J_0(2\pi c_{i,n}\nu)\right] \cos(2\pi\nu x)\ \mathrm{d}\nu. \tag{14}$$

Note that the density in Eq. (14) is independent of time. From Eq. (6), it follows that the mean of $\hat{\mu}_i(t)$ is constant and equal to zero, because

$$\begin{aligned}\hat{m}_{\mu_i} &= E\{\hat{\mu}_i(t)\}\\ &= \sum_{n=1}^{N_i} c_{i,n} E\{\cos(2\pi f_{i,n}t + \theta_{i,n})\}\\ &= 0.\end{aligned} \tag{15}$$

Substituting Eq. (6) in Eq. (11) results in the autocorrelation function

$$\hat{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N_i} \frac{c_{i,n}^2}{2} \cos(2\pi f_{i,n}\tau), \tag{16}$$

which is a function of $\tau = t_1 - t_2$. From Eqs. (14) to (16) it follows that $\hat{\mu}_i(t)$ is both FOS and WSS, because the first-order density $\hat{p}_{\mu_i}(x)$ is independent of time and the conditions (i) and (ii) (see Eqs. (10) and (11)) are fulfilled.

For a specific realization of the random phases $\theta_{i,n}$, it follows from Fig. 1 that the stochastic process $\hat{\mu}_i(t)$ results in a deterministic process (sample function) $\tilde{\mu}_i(t)$. The mean $\tilde{m}_{\mu_i}$ of $\tilde{\mu}_i(t)$ is obtained by substituting Eq. (7) in the right-hand side of Eq. (12). By taking into account that $f_{i,n} \neq 0$, we obtain

$$\tilde{m}_{\mu_i} = 0. \tag{17}$$

Comparing Eq. (15) with (17), we realize that the identity $\hat{m}_{\mu_i} = \tilde{m}_{\mu_i}$ holds, which states that $\hat{\mu}_i(t)$ is mean-ergodic. Substituting Eq. (7) into the right-hand side of Eq. (13) gives the autocorrelation function of $\tilde{\mu}_i(t)$ in the following form [6]

$$\hat{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N_i} \frac{c_{i,n}^2}{2} \cos(2\pi f_{i,n}\tau). \tag{18}$$

A comparison of Eqs. (16) and (18) shows that $\hat{\mu}_i(t)$ is autocorrelation ergodic, since the criterion $\hat{r}_{\mu_i\mu_i}(\tau) = \tilde{r}_{\mu_i\mu_i}(\tau)$ is fulfilled.

5.2. Class II Channel Simulators

Class II channel simulators are defined by the set of stochastic processes $\hat{\mu}_i(t)$ with constant gains $c_{i,n}$, random frequencies $\mathbf{f}_{i,n}$, and random phases $\theta_{i,n}$. For this class of channel simulators, we assume that the frequencies $\mathbf{f}_{i,n}$, are given by $\mathbf{f}_{i,n} = f_{\max}\sin(\mathbf{u}_{i,n}\pi/2)$, where $\mathbf{u}_{i,n}$ is a random generator having a uniform distribution in the interval (0, 1].

The assumption that the frequencies $\mathbf{f}_{i,n}$ are random variables has no effect on the density $\hat{p}_{\mu_i}(x)$ of $\hat{\mu}_i(t)$. Hence, the density $\hat{p}_{\mu_i}(x)$ is still given by Eq. (14). Also the mean $\hat{m}_{\mu_i}$ is equal to zero, and, thus, identical to Eq. (15). But the autocorrelation function $\hat{r}_{\mu_i\mu_i}(\tau)$ in Eq. (16) has to be averaged with respect to the random frequencies $\mathbf{f}_{i,n}$, i.e.,

$$\hat{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N_i} \frac{c_{i,n}^2}{2} E\{\cos(2\pi \mathbf{f}_{i,n}\tau)\}. \qquad (19)$$

Independent of a specific distribution of $\mathbf{f}_{i,n}$, it can be stated that $\hat{\mu}_i(t)$ is both FOS and WSS, since the density $\hat{p}_{\mu_i}(x)$ according to Eq. (14) is independent of time and the conditions in Eqs. (10) and (11) are fulfilled. Furthermore, the condition $\hat{m}_{\mu_i} = \tilde{m}_{\mu_i}$ holds, so that $\hat{\mu}_i(t)$ is mean-ergodic.

When using the Monte Carlo method [5,9], the gains $c_{i,n}$ and frequencies $\mathbf{f}_{i,n}$ are given by

$$c_{i,n} = \sigma_0\sqrt{\frac{2}{N_i}} \qquad \mathbf{f}_{i,n} = f_{\max}\sin\left(\frac{\pi}{2}\mathbf{u}_{i,n}\right), \qquad (20)$$

respectively, where $\mathbf{u}_{i,n}$ is a random variable with a uniform distribution in the interval (0, 1]. Using Eq. (20) in Eq. (19) results in

$$\hat{r}_{\mu_i\mu_i}(\tau) = \sigma_0^2 J_0(2\pi f_{\max}\tau). \qquad (21)$$

Obviously, the autocorrelation function $\hat{r}_{\mu_i\mu_i}(\tau)$ of the stochastic simulation model is identical to the autocorrelation function $r_{\mu_i\mu_i}(\tau)$ of the reference model described by Eq. (2). However, a comparison of Eqs. (21) and (18) shows that $\hat{r}_{\mu_i\mu_i}(\tau) \neq \tilde{r}_{\mu_i\mu_i}(\tau)$. Thus, the stochastic process $\hat{\mu}_i(t)$ of a Class II channel simulator is non-autocorrelation-ergodic. The problems of non-autocorrelation-ergodic channel simulators based on the Monte Carlo method have been discussed in [18].

5.3. Further Classes of Stochastic Channel Simulators

For any given number of sinusoids $N_i > 0$, the stochastic process $\hat{\mu}_i(t)$ depends on three types of parameters (gains, frequencies, and phases), each of which can be a collection of random variables or constants. However, at least one random variable is required to obtain a stochastic process $\hat{\mu}_i(t)$—otherwise a deterministic process $\tilde{\mu}_i(t)$ is obtained, as pointed out in Fig. 1. Therefore, all together $2^3 - 1 = 7$ classes of stochastic Rayleigh fading channel simulators can be defined. One class of stochastic channel simulators, for example, is defined by postulating random values for the gains $\mathbf{c}_{i,n}$, frequencies $\mathbf{f}_{i,n}$, and phases $\theta_{i,n}$. The analysis of the seven classes of stochastic channel simulators with respect to their stationary and ergodic properties is straightforward and leads to the results shown in Table I. The details are omitted here for reasons of brevity.

6. APPLICATION TO PARAMETER COMPUTATION METHODS

The above concept will now be applied to some selected parameter computation methods. Starting with the original Rice method [1,2] and using a limited number of sinusoids N_i, we realize by considering Eqs. (4) and (5) that the gains $c_{i,n}$ and frequencies $f_{i,n}$ are constant quantities. Due to the fact that the phases $\theta_{i,n}$, are random variables, it follows from Table I that the resulting channel simulator belongs to Class I. Such a channel simulator generates stochastic processes which are not only FOS but also mean- and autocorrelation-ergodic. On the other

Table I. Classes of Stochastic Channel Simulators and Their Statistical Properties

Class	Gains ($\mathbf{c}_{i,n}$)	Frequencies ($f_{i,n}$)	Phases ($\theta_{i,n}$)	First-order stationary	Wide-sense stationary	Mean-ergodic	Autocor.-ergodic
I	Const.	Const.	RV	Yes	Yes	Yes	Yes
II	Const.	RV	RV	Yes	Yes	Yes	No
III	Const.	RV	Const.	No/Yes[a, b, c, d]	No/Yes[a, b, c]	No/Yes[a, b]	No
IV	RV	Const.	Const.	No	No	No/Yes[e]	No
V	RV	Const.	RV	Yes	Yes	Yes	No
VI	RV	RV	Const.	No/Yes[a, b, c, d]	No/Yes[a, b, c]	No/Yes[e or a, b]	No
VII	RV	RV	RV	Yes	Yes	Yes	No

[a] If the density of $\mathbf{f}_{i,n}$ is an even function.
[b] If the boundary condition $\sum_{n=1}^{N_i}\cos(\theta_{i,n}) = 0$ is fulfilled.
[c] If the boundary condition $\sum_{n=1}^{N_i}\cos(2\theta_{i,n}) = 0$ is fulfilled.
[d] Only in the limit $t \to \pm\infty$.
[e] If the gains $\mathbf{c}_{i,n}$ have zero mean, i.e., $E\{\mathbf{c}_{i,n}\} = 0$.

Table II. Overview of Parameter Computation Methods and the Statistical Properties of the Resulting Channel Simulators

Parameter computation method	Class	First-order stationary	Wide-sense stationary	Mean-ergodic	Auto-correlation-ergodic
Rice method [1,2]	I	✔	✔	✔	✔
Monte Carlo method [5,9]	II	✔	✔	✔	✖
Jakes method (with random phases) [3]	I	✔	✔	✔	✔
Harmonic decomposition technique [14]	I	✔	✔	✔	✔
Method of equal distances [10]	I	✔	✔	✔	✔
Method of equal areas [10]	I	✔	✔	✔	✔
Mean-square-error method [10]	I	✔	✔	✔	✔
Method of exact Doppler spread [4]	I	✔	✔	✔	✔
L_p-norm method [4]	I	✔	✔	✔	✔
Method proposed by Zheng and Xiao [11]	II	✔	✔	✔	✖
Improved method by Zheng and Xiao [12]	VII	✔	✔	✔	✖

hand, if the Monte Carlo method [5,9] or the method due to Zheng and Xiao [11] is applied, then the gains $c_{i,n}$ are constant quantities, and the frequencies $\mathbf{f}_{i,n}$ and phases $\theta_{i,n}$ are random variables. Consequently, the designed channel simulator can be identified as a Class II channel simulator, which is FOS, WSS, and mean-ergodic but unfortunately non-autocorrelation-ergodic. For a given parameter computation method, the stationary and ergodic properties of the designed channel simulator can easily be checked with the help of Table I. Note that the above concept can be applied to any given parameter computation method. For some selected methods, the obtained results are presented in Table II.

Of great popularity is the Jakes method [3]. When this method is used, then the gains $c_{i,n}$, frequencies $f_{i,n}$, and phases $\theta_{i,n}$ are constant quantities. Consequently, the Jakes channel simulator is per definition deterministic. To obtain a stochastic channel simulator, it is advisable to replace the deterministic phases[2] $\theta_{i,n}$ by random phases $\theta_{i,n}$. In this case, the Jakes method results in a Class I channel simulator, which is FOS, WSS, mean-ergodic, and auto-correlation-ergodic. However, this is in contrast to the analysis of Jakes' simulator in [13], where it has been assumed that the gains $c_{i,n}$ are random variables. But this assumption is not in the sense of the original Jakes method and leads not to the correct results.

Nevertheless, the Jakes simulator has some disadvantages [19], which can be avoided, e.g., by using the method of exact Doppler spread [4] or the even more powerful L_p-norm method [4]. Both methods enable the design of deterministic channel simulators, where all parameters are fixed including the phases $\theta_{i,n}$. The investigation of the stationary and ergodic properties of deterministic processes makes no sense, since the concept of stationarity and ergodicity is only applicable to stochastic processes. A deterministic channel simulator can be interpreted as an emulator for sample functions of the underlying stochastic channel simulator. According to Fig. 1, the corresponding stochastic simulation model is obtained by replacing the constant phases $\theta_{i,n}$ by random phases $\theta_{i,n}$. Important is now to realize that all stochastic channel simulators derived in this way from deterministic channel simulators are Class I channel simulators with the known statistical properties as they are listed in Table I. This must be taken into account when considering the results shown in Table II.

7. CONCLUSION

In this paper, the stationary and ergodic properties of Rayleigh fading channel simulators using the sum-of-sinusoids principle have been analyzed. Depending on whether the model parameters are random variables or constant quantities, altogether seven classes of stochastic channel simulators have been defined. It turned out that if and only if the phases are random variables and the other model parameters are constant, then the channel simulator is both stationary and ergodic. If the frequencies are random variables, then the stochastic channel simulator is stationary but non-autocorrelation-ergodic. The worst case however, is given when the gains are random variables and the other parameters are constant. Then, a non-stationary stochastic channel simulator is obtained.

[2] It should be noted that the phases $\theta_{i,n}$ in Jakes' channel simulator are equal to 0 for all $i = 1, 2$ and $n = 1, 2, \ldots, N_i$ [19].

The results presented here are of fundamental importance for the performance evaluation of existing design methods. Moreover, the results give strategic guidelines to engineers for the development of new design methods.

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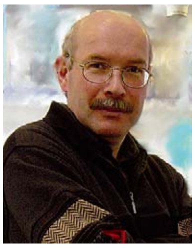

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