On the Statistical Properties of Deterministic Simulation Models for Mobile Fading Channels

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Abstract—Rice’s sum of sinusoids can be used for an efficient approximation of colored Gaussian noise processes and is therefore of great importance to the software and hardware realization of mobile fading channel models. Although several methods can be found in the literature for the computation of the parameters characterizing a proper set of sinusoids, only less is reported about the statistical properties of the resulting (deterministic) simulation model. This is the topic of the present paper, whereby not only the simulation model’s amplitude and phase probability density function (pdf) will be investigated, but also higher order statistics [e.g., level-crossing rate (lcr) and average duration of fades (adf’s)]. It will be shown that due to the deterministic nature of the simulation model, analytical expressions for the pdf of amplitude and phase, autocorrelation function (acf), cross-correlation function (ccf), lcr, and adf’s can be derived. We also propose a new procedure for the determination of an optimal set of sinusoids, i.e., the method results for a given number of sinusoids in an optimal approximation of Gaussian, Rayleigh, and Rice processes with given Doppler power spectral density (psd) properties. It will be shown that the new method can easily be applied to the approximation of various other kinds of distribution functions, such as the Nakagami and Weibull distributions. Moreover, a quasi-optimal parameter computation method is presented.

Index Terms—Deterministic simulation models, nonfrequency-selective fading channels, Rician channels, statistics.

I. INTRODUCTION

In general, simulation models for mobile radio channels are realized by employing at least two or more colored Gaussian noise processes. For instance, for the realization of a Rayleigh or Rice process, two real colored Gaussian noise processes are required, whereas the realization of a Suzuki process [1], which is a product process of a Rayleigh and a lognormal process, is based on three real colored Gaussian noise processes. Such processes (Rayleigh, Rice, and Suzuki) are often used as appropriate stochastic models for describing the fading behavior of the envelope of the signal received over land mobile radio channels, which are often classified as frequency-nonselective channels. A further example is given by modeling a (γ-f-path) frequency-selective mobile radio propagation channel by using the n-tap delay line model [2]. This requires the realization of 2γ real colored Gaussian noise processes. All these examples show us that an efficient design method for the realization of colored Gaussian noise processes is of particular importance in the area of mobile radio channel modeling.

A well-known method for the design of colored Gaussian noise processes is to shape a white Gaussian noise (WGN) process by means of a filter that has a transfer function, which is equal to the square root of the Doppler power spectral density (psd) of the fading process [see Fig. 1(a)]. Another method, which is recently coming more and more into use, is based on Rice’s sum of sinusoids [3], [4]. In this case, a colored Gaussian noise process is approximated by a finite sum of weighted and properly designed sinusoids. The resulting signal flow diagram of such an approximated Gaussian noise process is shown in Fig. 1(b), where the quantities $c_n$, $f_n$, and $\theta_n$ are called Doppler coefficients, discrete Doppler frequencies, and Doppler phases, respectively, and $N$ denotes the number of sinusoids.

In the meanwhile, diverse methods have been developed for the derivation of the relevant model parameters (Doppler coefficients $c_n$ and discrete Doppler frequencies $f_n$), for example, the method of equal distances [5], [6] and the mean-square-error method [6]. The characteristic of both methods is just the same as Rice’s original method [3], [4], namely, the difference between two adjacent discrete Doppler frequencies is equidistant, but the Doppler coefficients are adapted to a given Doppler psd in a method-specific manner. The disadvantage of these methods is that (due to the equidistant discrete Doppler frequencies) the approximated colored Gaussian noise process $\hat{\mu}(t)$ is periodical. Another method is known as the method of equal areas [5], [6]. That procedure results in a relatively satisfactory approximation of the desired statistics for the Jakes Doppler psd, even for a small number of sinusoids, but fails or requires a comparatively large number of
sinusoids for other types of Doppler power spectral densities such as those with Gaussian shapes. Widespread in use is the so-called Monte Carlo method [7], [8]. But unfortunately the performance of that procedure is poor [9]. The reason is that the discrete Doppler frequencies are random variables, and, therefore, the moments of the Doppler psd of the designed Gaussian noise process \( \hat{p}(t) \) are also random variables, which can differ widely from the desired (ideal) moments, even for a large number of sinusoids \( N \), say \( N = 100 \) [6], [10].

A further design method was derived by Jakes [11]. For the Jakes method, an efficient hardware implementation, using the Intel 8088 microprocessor, is described in [12].

Although all the parameter computation methods can be classified in deterministic and statistic procedures, the resulting simulation model is in any case a deterministic one because all parameters of the simulation model have to be determined during the simulation setup phase, and, thus, they are known and constant quantities during the total simulation run phase. In this paper, the statistical properties of such deterministic simulation models will be investigated.

The paper recapitulates in Section II the statistical properties of Rice processes with given spectral characteristics for the underlying Gaussian processes. Section III introduces the concept of deterministic simulation models and discusses the performance of three different parameter computation methods, whereby two of them are new. One of the new procedures is for a given number of sinusoids optimal in the following sense. First, the Doppler coefficients \( c_n \) are calculated such that the probability density of the approximated colored Gaussian process \( \hat{p}(t) \) approximates optimally the aspired Gaussian probability density function (pdf). Second, the discrete Doppler frequencies \( f_n \) are optimized so that the autocorrelation function (acf) of \( \hat{p}(t) \) approximates optimally the desired acf of a given colored Gaussian noise process. Moreover, analytical expressions for the deterministic simulation model’s amplitude and phase pdf, acf, and cross-correlation function (ccf) will be derived. Section IV investigates higher order statistical properties of deterministic simulation models such as the level-crossing rate (lcr), average duration of fades (adf’s), and the pdf of the fading intervals. It is shown analytically and experimentally that the lcr of properly designed and dimensioned deterministic simulation models for Rayleigh and Rice processes is extremely close to the theory, even if the number of sinusoids is small, say seven or eight. Section V concludes the paper.

II. A STOCHASTIC ANALYTICAL MODEL FOR RICE PROCESSES

In order to compare the statistical properties of the deterministic simulation model, as will be introduced in Section III with the desired (exact) statistics, a reference model is required. As a reference model, we consider in this section an analytical model for Rice processes. In cases where the influence of a direct line of sight component cannot be neglected, Rice processes are often used as suitable stochastical models for characterizing the fading behavior of the received signal level of land mobile terrestrial channels as well as land mobile satellite channels.

A. Description of the Analytical Model

A Rice process, \( \xi(t) \), is defined by taking the absolute value of a nonzero mean complex Gaussian process

\[
\mu_p(t) = \mu(t) + m(t)
\]

according to

\[
\xi(t) = \lfloor |\mu_p(t)| \rfloor.
\]

In (1), all the scattered components in the received signal are represented by a zero-mean complex Gaussian noise process

\[
\mu(t) = \mu_1(t) + j \mu_2(t)
\]

with uncorrelated real components \( \mu_i(t) \), \( i = 1, 2 \), and variance \( \text{Var} \{ \mu_i(t) \} = 2 \text{Var} \{ \mu_2(t) \} = 2 \sigma_\mu^2 \), whereas the influence of a direct (line of sight) component is taken into account here by a time-variant mean value of the form

\[
m(t) = m_1(t) + j m_2(t) = \rho e^{j(2\pi f_D t + \theta_\mu)}.
\]

Amplitude, Doppler frequency, and phase of the direct component are denoted by \( \rho \), \( f_D \), and \( \theta_\mu \), respectively. Observe that for \( f_D = 0 \), the mean value \( m_i(t) = m = \rho e^{j\theta_\mu} \) is obviously time invariant, which directly corresponds to an orthogonality between the direction of the incoming direct wave component at the receiving antenna of the vehicle and the direction of the vehicle movement. Throughout this paper, we consider the more general case where \( f_D \neq 0 \), and, hence, the direct component \( m(t) \) depends on time.

A typical and often-assumed shape for the Doppler psd of the complex Gaussian noise process \( \mu(t) \), \( S_{\mu\mu}(f) \), is for mobile fading channel models given by the Jakes psd [11]

\[
S_{\mu\mu}(f) = \begin{cases} \frac{2\sigma_\mu^2}{\pi f_{\text{max}} \sqrt{1-(f/f_{\text{max}})^2}}, & |f| \leq f_{\text{max}} \\ 0, & |f| > f_{\text{max}} \end{cases}
\]

where \( f_{\text{max}} \) is the maximum Doppler frequency. From this equation, it follows for the corresponding acf \( r_{\mu\mu}(t) \), i.e., the inverse Fourier transform of \( S_{\mu\mu}(f) \), the following relation:

\[
r_{\mu\mu}(t) = 2\sigma_\mu^2 J_0(2\pi f_{\text{max}} t)
\]

where \( J_0(\cdot) \) denotes the zeroth-order Bessel function of the first kind. The resulting structure of the analytical model for Rice processes \( \xi(t) \) with underlying Jakes psd characteristic is shown in Fig. 2(a).

Another important shape for the Doppler psd \( S_{\mu\mu}(f) \) is given by the frequency-shifted Gaussian psd

\[
S_{\mu\mu}(f) = \frac{2\sigma_\mu^2}{f_c} \sqrt{\frac{\ln 2}{\pi}} e^{-\left(\frac{f-f_c}{\sqrt{2} \sigma_\mu f_c}\right)^2}
\]

where \( f_s \) and \( f_c \) are the frequency shift and 3-dB cutoff frequency, respectively. Superpositions of two Gaussian psd functions with different values for \( \sigma_\mu^2 \), \( f_s \), and \( f_c \) have been specified, for example, in [13] for describing the typical Doppler psd of large echo delays, which occur in the frequency-selective mobile radio channel of the pan-European cellular global system for mobile communications (GSM). Furthermore, frequency-nonshifted Gaussian psd functions,
i.e., \( f_s = 0 \) in (7), have been introduced for describing the Doppler psd of aeronautical channels [14], [15]. The acf of the Gaussian psd \( S_{\mu \mu}(f) \) is given by

\[
\rho_{\mu \mu}(t) = 2\alpha_0^2 e^{-\left(\pi \frac{t^2}{\alpha_0^2}\right)^2} e^{j2\pi f_0 t}.
\]  (8)

Observe that the acf \( \rho_{\mu \mu}(t) \) is complex if \( f_s \neq 0 \). Thus, it follows from the properties of the Fourier transform that in such cases the real Gaussian processes \( \mu_1(t) \) and \( \mu_2(t) \) are correlated. Fig. a list2(b) shows the resulting structure of the analytical model for Rice processes \( \xi(t) \) with an underlying frequency-shifted Gaussian psd characteristic.

**B. Statistical Properties of the Analytical Model**

In this section, we study besides the amplitude and phase pdf, the lcr and the adf’s of Rice processes with given spectral shapes for the underlying Gaussian processes. We refer to these (ideal) statistical properties in Section III, where we investigate the statistics of deterministic simulation models.

The pdf of the Rice process \( \xi(t) \), \( p_{\xi}(x) \), is given by [16]

\[
p_{\xi}(x) = \begin{cases} 
\frac{x}{\sigma_0^2} e^{-\frac{x^2}{2\sigma_0^2}} I_0 \left( \frac{x \rho}{\sigma_0} \right), & x \geq 0 \\
0, & x < 0
\end{cases}
\]  (9)

where \( I_0(\cdot) \) designates the zeroth-order modified Bessel function of the first kind and \( \psi_0 \) represents the mean power of the Gaussian process \( \mu_1(t) \), i.e., \( \psi_0 = r_{\mu_1\mu_1}(0) = r_{\mu_\mu}(0)/2, \ i = 1, 2 \). From (6) and (8), it follows that \( \psi_0 \) is for the Jakes and Gaussian psd identical with \( \psi_0 = \sigma_0^2 f_0^2 \).

Obviously, the exact form of the Doppler psd of \( \mu(t) \) has no influence on the behavior of the pdf \( p_{\xi}(x) \). Such an independence of the Doppler psd \( S_{\mu \mu}(f) \) holds also true for the phase pdf of the complex Gaussian process \( \mu(t) \). \( p_{\phi}(\theta) \), which is given by [17]

\[
p_{\phi}(\theta) = p_{\phi}(\theta; \psi_0) = e^{-\frac{2\theta^2}{2\psi_0}} \left( 1 + \sqrt{\frac{n}{2\psi_0}} \rho \right) \times \cos(\theta - 2\pi f_0 t - \theta_0) e^{\frac{\rho^2 \cos^2(\theta - 2\pi f_0 t - \theta_0)}{2n}} \times \left[ 1 + \text{erf} \left( \frac{\rho \cos(\theta - 2\pi f_0 t - \theta_0)}{\sqrt{2n}} \right) \right]
\]  (10)

where \( \text{erf}(\cdot) \) is the error function and \( \theta \) is within the interval \( [0, 2\pi] \).

By taking notice of (9) and (10), we see that in the limit for \( \rho \to 0 \), the amplitude pdf \( p_{\xi}(x) \) tends to the Rayleigh distribution

\[
p_{\xi}(x) = \begin{cases} 
\frac{x}{\sigma_0^2} e^{-\frac{x^2}{2\sigma_0^2}} I_0 \left( \frac{x \rho}{\sigma_0} \right), & x \geq 0 \\
0, & x < 0
\end{cases}
\]  (11)
whereas the phase pdf $p_{0}(\theta)$ tends to the uniform distribution $p_{0}(\theta) = \frac{1}{2\pi}$, $\theta \in [0,2\pi)$, respectively.

The lcr, denoted here by $N_{\xi}(r)$, provides us with a measure of the average number of crossings per second at which the envelope $\xi(t) = |\mu(t)|$ crosses a specified signal level $r$ with positive slope.

We quote for the lcr $N_{\xi}(r)$ the result from [17], where $N_{\xi}(r)$ has been derived for Rice processes with cross-correlated inphase and quadrature components

$$N_{\xi}(r) = \frac{r\sqrt{\beta}}{\pi\sigma_{0}^{2}} e^{-\frac{\beta}{2\sigma_{0}^{2}}} \int_{0}^{\pi/2} \cosh\left(\frac{r\rho}{\sigma_{0}}\cos\theta\right) \times \left\{ e^{-\left(\alpha_{0}\sin\theta\right)^{2}} + \sqrt{\pi} \alpha_{0} \sin\theta \right\} d\theta$$

(12)

where the quantities $\alpha_{0}$ and $\beta$ are given by

$$\alpha = \left(2\pi f_{p} - \frac{\phi_{0}}{\psi_{0}}\right) / \sqrt{2\beta}$$

(13)

$$\beta = -\psi_{0} - \frac{\phi_{0}^{2}}{\psi_{0}}$$

(14)

Thereby, the above notations $\psi_{0}$ and $\phi_{0}$ have been introduced for denoting the curvature of the acf $r_{\mu_{1}\mu_{2}}(t)$ and the gradient of the ccf $r_{\mu_{1}\mu_{2}}(t)$ at $t = 0$, i.e.,

$$\psi_{0} = \left. \frac{d^{2}}{dt^{2}} r_{\mu_{1}\mu_{2}}(t) \right|_{t=0} = \dot{\psi}_{0}$$

(15)

$$\phi_{0} = \left. \frac{d}{dt} r_{\mu_{1}\mu_{2}}(t) \right|_{t=0} = \dot{\phi}_{0}$$

(16)

respectively. If the Doppler frequency of the direct component $f_{p}$ is zero and if the real Gaussian noise processes $\mu_{1}(t)$ and $\mu_{2}(t)$ are uncorrelated, i.e., $\phi_{0} = 0$, then it follows from (13) that $\alpha = 0$ and, consequently, the lcr $N_{\xi}(r)$ according to (12) reduces to the expression

$$N_{\xi}(r) = \sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2\sigma_{0}^{2}}} I_{0}\left(\frac{r\rho}{\psi_{0}}\right) = \sqrt{\frac{\beta}{2\pi}} \cdot p_{\xi}(r)$$

(17)

where $\beta = -\psi_{0} \geq 0$. In such cases, the lcr $N_{\xi}(r)$ is simply the Rice distribution $p_{\xi}(r)$ multiplied by a constant factor $\sqrt{\frac{\beta}{2\pi}}$, which represents the influence of the curvature of the acf $r_{\mu_{1}\mu_{1}}(t)$ at $t = 0$.

The adf's, denoted here by $T_{\xi}(r)$, is the mean value for the length of all time intervals over which the signal envelope $\xi(t) = |\mu(t)|$ is below a specified level $r$. The adf's $T_{\xi}(r)$ is defined by (c.f. [11, p. 36])

$$T_{\xi}(r) = \frac{P_{\xi}(r)}{N_{\xi}(r)}$$

(18)

where $P_{\xi}(r)$ designates the probability that the signal envelope $\xi(t)$ is found below the level $r$, i.e.,

$$P_{\xi}(r) = \Pr\{\xi(t) \leq r\} = \int_{0}^{r} p_{\xi}(x) dx = e^{-\frac{\beta}{2\sigma_{0}^{2}}} \int_{0}^{r} xe^{-\frac{x^{2}}{2\sigma_{0}^{2}}} I_{0}\left(\frac{x\rho}{\psi_{0}}\right) dx.$$

(19)

Equations (12) and (18) show us that the lcr $N_{\xi}(r)$ and the adf's $T_{\xi}(r)$ are both functions of the characteristic quantities $\alpha_{0}$ and $\beta$ [see (13) and (14)], which are completely defined by the Doppler psd $S_{\mu}(f)$ and the Doppler frequency of the direct component $f_{p}$. For the Jakes and frequency-shifted Gaussian psd, we obtain for $\alpha_{0}$ and $\beta$—by using (6), (8), and (13)–(16) in connection with the relation $r_{\mu\mu}(t) = 2(r_{\mu_{1}\mu_{1}}(t) + \dot{r}_{\mu_{1}\mu_{1}}(t))$—the following result:

$$\alpha = \frac{1}{\sqrt{\sigma_{0}^{2}}} \cdot f_{p} \left. f_{\mu}\right|_{f_{\mu}=\sqrt{\ln 2}}$$

(20a)

$$\beta = \frac{2\psi_{0}(\pi f_{p\mu\max})^{2}}{2\psi_{0}(\pi f_{p\mu\max})^{2}}.$$

(20b)

where $\psi_{0} = \sigma_{0}^{2}$.

From the above, it is clear now that the higher order statistics considered here (lcr $N_{\xi}(r)$ and adf's $T_{\xi}(r)$) are completely determined by the amplitude and Doppler frequency of the direct component, i.e., $\rho$ and $f_{p}$, as well as by the shape of the acf $r_{\mu\mu}(t)$ and its time derivative of first and second order at $t = 0$.

III. A DETERMINISTIC SIMULATION MODEL FOR RICE PROCESSES

In this section, we investigate the statistical properties of deterministic simulation models for Rice processes with desired spectral characteristics for the underlying Gaussian processes. Therefore, we first of all introduce the reader to the concept of deterministic simulation models, and, subsequently, we present a new and optimal method for the computation of the simulation model parameters. For reasons of comparison, we also investigate the statistical properties of the simulation system for two other parameter computation methods.

Let us proceed by considering the two real functions $\tilde{\mu}_{1}(t)$ and $\tilde{\mu}_{2}(t)$, which will be expressed as follows:

$$\tilde{\mu}_{i}(t) = \sum_{n=1}^{N_{i}} c_{i\mu} \cos(2\pi f_{i\mu} t + \theta_{i\mu}), \quad i = 1,2$$

(22)

where $N_{i}$ denotes the number of sinusoids of the function $\tilde{\mu}_{i}(t)$. The quantities $c_{i\mu}$, $f_{i\mu}$, and $\theta_{i\mu}$ are simulation model parameters, which are adapted to the desired Doppler psd function and are therefore called Doppler coefficients, discrete Doppler frequencies, and Doppler phases, respectively. It is worth mentioning that the simulation model parameters $c_{i\mu}$, $f_{i\mu}$, and $\theta_{i\mu}$ have to be computed during the simulation setup phase, e.g., by one of the methods described in the following sections. Afterwards, these parameters are known quantities and are kept constant during the whole simulation run phase.

From the fact that all parameters are known quantities, it follows that $\tilde{\mu}_{i}(t)$ itself is determined for all time and thus $\tilde{\mu}_{i}(t)$ can be considered as a deterministic function. We will see that the statistical properties of $\tilde{\mu}_{i}(t)$ approximate closely the statistical properties of colored Gaussian stochastic processes. In order to emphasize that property, we will denote henceforth...
the deterministic function $\tilde{\mu}(t)$ as a deterministic (Gaussian) process.

Consequently, the interpretation of $\tilde{\mu}(t)$ as a deterministic process (function) allows us to compute an analytical expression for the corresponding acf $\tilde{\gamma}_{\mu}(t)$ via the definition

$$\tilde{\gamma}_{\mu}(t) := \lim_{T_0 \to \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} \tilde{\mu}(\tau)\tilde{\mu}(t+\tau)d\tau.$$  \hspace{1cm} (23)

Substituting (22) in (23) and taking thereupon the Fourier transform, we find the following analytical expressions for the acf $\tilde{\gamma}_{\mu}(t)$ and the corresponding psd $\tilde{S}_{\mu}(f)$:

$$\tilde{\gamma}_{\mu}(t) = \sum_{n=1}^{N} \frac{c_{2n}}{2} \cos(2\pi f_{2n}t)$$  \hspace{1cm} (24a)

$$\tilde{S}_{\mu}(f) = \sum_{n=1}^{N} \frac{c_{2n}}{4} \left[ \delta(f - f_{2n}) + \delta(f + f_{2n}) \right]$$  \hspace{1cm} (24b)

for $i = 1, 2$.

Notice that the deterministic processes $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ are uncorrelated if and only if $f_{1,n} \neq \pm f_{2,m}$ for all $n = 1, 2, \ldots, N_1$ and $m = 1, 2, \ldots, N_2$. But if $f_{1,n} = \pm f_{2,m}$ is valid for some or all $n, m$, then $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ are correlated, and the following cross-correlation function $\tilde{\gamma}_{\mu_1,\mu_2}(t)$ can be derived, as given in (25) at the bottom of the page, where $N$ indicates the minimum value of $N_1$ and $N_2$, i.e., $N = \min\{N_1, N_2\}$.

Fig. 3 shows the resulting general structure of the deterministic simulation model for Rice processes in its continuous-time representation. From that, the discrete-time fading simulation model can immediately be obtained by replacing the time variable $t$ by $t = kT$, where $k$ is an integer and $T$ denotes the sampling interval.

For the parameters $c_{i,n}$ and $f_{i,n}$ appearing in (22), we will present in the next two sections a new computation procedure, which is for a given number of sinusoids $N_2$ optimal in the following two senses.

$$\tilde{\gamma}_{\mu_1,\mu_2}(t) = \begin{cases} \sum_{n=1}^{N} \frac{c_{2n}}{2} \cos(2\pi f_{2n}t - \theta_{2n} \pm \theta_{2m}), & \text{if } f_{1,n} = \pm f_{2,m} \\ 0, & \text{if } f_{1,n} \neq \pm f_{2,m} \end{cases}$$  \hspace{1cm} (25)
1) The pdf of the deterministic process \( \hat{\mu}(t) \), named by \( \hat{\mu}_r(x) \), is according to the \( L_2 \) norm, an optimal approximation of the ideal (desired) pdf \( p_{\mu}(x) \) of the stochastic process \( \mu(t) \).

2) The acf of the deterministic process \( \hat{\mu}(t) \), named by \( \hat{\rho}_{\mu}(t) \), is according to the \( L_2 \) norm, an optimal approximation of the ideal (desired) acf \( \rho_{\mu}(t) \) of the stochastic process \( \mu(t) \) within a distinct time interval.

A. Determination of Optimal Doppler Coefficients

In this section, we compute the Doppler coefficients \( \{ c_{\text{in}} \} \) in such a way that the pdf \( \hat{\mu}_r(x) \) of the deterministic process \( \hat{\mu}(t) \) results for a given number of sinusoids \( N_i \) in an optimal approximation of the ideal Gaussian pdf \( p_{\mu}(x) \) of the stochastic process \( \mu(t) \). In the first step, we derive the pdf of \( \hat{\mu}_r(t) \) [see (22)]. For that purpose, let us consider a single sinusoid

\[
\hat{\mu}_{i,n}(t) = c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}).
\]

Furthermore, let us assume in this section that the time \( t \) is a random variable, which is uniformly distributed in the interval \((0, f_{i,n}^{-1})\) (suppose \( f_{i,n} \neq 0 \)). Then, \( \hat{\mu}_{i,n}(t) \) is also a random variable, and the pdf of \( \hat{\mu}_{i,n}(t) \) is given by (c.f. [16, p. 98])

\[
\hat{\mu}_{i,n}(x) = \begin{cases} \frac{1}{\pi c_{i,n} \sqrt{1 - (x/c_{i,n})^2}} & |x| < c_{i,n} \\ 0 & |x| \geq c_{i,n} \end{cases}
\]

If the random variables \( \hat{\mu}_{i,n}(t) \) are independent with respective densities \( \hat{\mu}_{i,n}(x) \), then the density \( \hat{\mu}_r(x) \) of their sum

\[
\hat{\mu}_r(t) = \hat{\mu}_{i,1}(t) + \hat{\mu}_{i,2}(t) + \cdots + \hat{\mu}_{i,N_i}(t)
\]

equals the convolution of their corresponding densities

\[
\hat{\mu}_r(x) = \hat{\mu}_{i,1}(x) \ast \hat{\mu}_{i,2}(x) \ast \cdots \ast \hat{\mu}_{i,N_i}(x).
\]

Although (29) provides us with a rule for the computation of the density \( \hat{\mu}_r(x) \), it is more appropriate to apply the concept of the characteristic function.1

The characteristic function \( \hat{\psi}_{\mu_{i,n}}(\nu) \) of the random variable \( \hat{\mu}_{i,n}(t) \) is obtained by taking the Fourier transform of (27)

\[
\hat{\psi}_{\mu_{i,n}}(\nu) = \int_{-\infty}^{\infty} \hat{\mu}_{i,n}(x) e^{-2\pi i \nu x} dx.
\]

The properties of characteristic functions are essentially the same as the properties of Fourier transforms. Thus, the convolution of the densities (29) reduces to the product of their characteristic functions, i.e.,

\[
\hat{\psi}_{\mu}(\nu) = \hat{\psi}_{\mu_{i,1}}(\nu) \cdot \hat{\psi}_{\mu_{i,2}}(\nu) \cdots \hat{\psi}_{\mu_{i,N_i}}(\nu)
\]

\[
= \prod_{n=1}^{N_i} \int_{-\infty}^{\infty} \hat{\psi}_{\mu_{i,n}}(\nu) e^{2\pi i \nu x} dx.
\]

1 The characteristic function of a random variable \( X \) is defined as the statistical average \( \Psi(\nu) = E(e^{2\pi i \nu X}) = \int_{-\infty}^{\infty} e^{2\pi i \nu x} p_X(x) dx \), where \( \nu \) is a real variable. In spite of the positive sign in the exponential, \( \Psi(\nu) \) is often called the Fourier transform of \( p_X(x) \).

Finally, from the above equation, the pdf \( \hat{\mu}_r(x) \) can be obtained by taking the inverse Fourier transform, i.e.,

\[
\hat{\mu}_r(x) = \int_{-\infty}^{\infty} \hat{\psi}_{\mu}(\nu) e^{-2\pi i \nu x} d\nu = 2 \int_{0}^{\infty} \prod_{n=1}^{N_i} J_0(2\pi c_{i,n} \nu) \cos(2\pi \nu x) d\nu.
\]

Let \( f_{i,n} \neq 0 \) and \( c_{i,n} = \sigma_0 \sqrt{2/N_i} \) for all \( n = 1, 2, \ldots, N_i \) and \( i = 1, 2 \), then the sum \( \hat{\mu}(t) \) [defined by (28)] is a random variable with zero mean and variance \( \sigma^2_0 = (c_0^2 + \cdots + c_{N_i}^2)/2 = \sigma_0^2 \).

The central limit theorem states that in the limit \( N_i \to \infty \), the random variable \( \hat{\mu}(t) \) tends to a Gaussian-distributed random variable \( \mu(t) \) with zero mean and variance \( \sigma_0^2 \), i.e.,

\[
\lim_{N_i \to \infty} \hat{\mu}_r(x) = \mu_r(x) = \frac{1}{\sqrt{2\pi \sigma_0^2}} e^{-2\pi x^2/(2\sigma_0^2)}.
\]

By taking the Fourier transform of the equation given above, we obtain

\[
\lim_{N_i \to \infty} \hat{\mu}_r(x) = \hat{\mu}(x) = \frac{1}{\sqrt{2\pi \sigma_0^2}} e^{-2\pi x^2/(2\sigma_0^2)}.
\]

Of course, for a limited number of sinusoids \( N_i \), we must write \( \hat{\mu}_r(x) \approx \mu_r(x) \) and \( \hat{\mu}(x) \approx \mu(x) \). By considering Fig. 4(a), where \( \hat{\mu}_r(x) \) has been evaluated for \( \hat{\mu}_r(x) \), we see that the approximation error is small and can be neglected for most practical applications if \( N_i \geq 7 \).

Now, the following question arises. Does there exist an optimal set of Doppler coefficients \( \{ c_{i,n} \} \) in such a sense that the pdf of the simulation system \( \hat{\mu}_r(x) \) results for a given number of sinusoids \( N_i \) in an optimal approximation of the ideal Gaussian pdf \( p_{\mu}(x) \)? This question can be answered by considering the following \( L_2 \) norm, which is also known as the square root of the mean square error, i.e.,

\[
E_{\mu}^{(2)} = \left( \int_{-\infty}^{\infty} |\hat{\mu}_r(x) - \mu_r(x)|^2 dx \right)^{1/2}.
\]

After substituting (32) and (33) in (36), the above error function can be minimized numerically by using a suitable optimization procedure such as the method described in [18]. The minimization of (36) gives us an optimized set for the Doppler coefficients \( \{ c_{\text{opt}} \} \). Fig. 4(b) shows the resulting pdf of the simulation system \( \hat{\mu}_r(x) \) by using the optimized quantities \( c_{\text{opt}} \). A proper choice of the initial values of the Doppler coefficients is given by choosing \( c_{\text{opt}} = \sigma_0 \sqrt{2/N_i} \). For these values, the evaluation of the error function \( E_{\mu}^{(2)} \) is shown in Fig. 5(a). The same figure shows also the resulting error function \( E_{\mu}^{(2)} \) for the optimized Doppler coefficients \( c_{\text{opt}} \), which are for a given number of sinusoids \( N_i \), all identical after the minimization of (36). A further result worth mentioning is that the optimization procedure always decreases the initial values for the Doppler coefficients, i.e., \( c_{\text{opt}} \leq c_{\text{opt}} \).
Fig. 4. The approximated Gaussian pdf $\hat{p}_{\mu}(x)$ for (a) $c_{i,n} = \sigma_0 \sqrt{2/N_i}$ and (b) $c_{i,n} = c_{i,n}^{(\text{opt})}$ for $N_i = 5, 7, \infty$ and $\sigma_0^2 = 1$.

Fig. 5. (a) Error function $E^{(2)}_{\hat{p}_\mu}$ and (b) variance $\sigma_n^2$ of the deterministic process $\hat{p}_\mu(t)$ for $c_{i,n} = \sigma_0 \sqrt{2/N_i}$ (ooo) and $c_{i,n} = c_{i,n}^{(\text{opt})}$ (***$||$) with $\sigma_0^2 = 1$.

$\sigma_0 \sqrt{2/N_i}$, and therefore, the variance of the deterministic process $\hat{\mu}_i(t)$ is for finite $N_i$ always smaller than the variance of the stochastic process $\mu_i(t)$ as depicted in Fig. 5(b). But this effect diminishes if $N_i$ increases and can completely be neglected for reasonably values of $N_i$, say $N_i \geq 7$. The consequence from the above is that the improvements made by the optimization procedure are negligible for most practical applications if $N_i \geq 7$, and, thus, we can say for simplicity that $c_{i,n}^{(\text{opt})}$ is approximately given by

$$c_{i,n} = \sigma_0 \sqrt{2/N_i}, \quad \text{if} \quad N_i \geq 7 \quad (37)$$

for all $n = 1, 2, \ldots, N_i$.

Next, let us impose on this optimization approach the power constraint, i.e., $\sigma_0^2 = \sigma_0^2$. The power constraint is nothing else but a boundary condition, which can easily be included in the optimization procedure by optimizing the first $N_i - 1$ Doppler coefficients $c_{1,n}, \ldots, c_{i-1,n}$, whereas the remaining Doppler coefficient $c_{i,n}$ will be computed so that $\sigma_0^2 = \sigma_0^2$ yields. It follows that the result (after applying the optimization procedure) is for all $N_i$ identical with (37), even if the initial values are chosen arbitrarily. Thus, (37) ensures, for a given number of sinusoids $N_i$, an optimal approximation of ideal Gaussian pdf’s $p_{\mu_i}(x_i)$ given the power constraint. Henceforth, we will make use of (37) if we consider the approximation of stochastic processes with Gaussian distribution.

In the following, we are interested in the amplitude and phase pdf of the deterministic simulation system for Rice processes as represented in Fig. 3. We only consider the time-invariant direct component $m = m_1 + jm_2$ [see (4) with $f_0 = 0$], where the pdf of the real and imaginary part are given by $p_{m_i}(x_i) = \delta(x_i - m_i)$, $i = 1, 2$. Thus, the pdf of the deterministic process $\hat{p}_{\mu_i}(t)$ can be expressed by

$$\hat{p}_{\mu_i}(x_i) = \hat{p}_{\mu_1}(x_i) * p_{m_i}(x_i) = \hat{p}_{\mu_1}(x_i - m_i) \quad (38)$$

Let us assume that the deterministic processes $\hat{p}_{\mu_1}(t)$ and $\hat{p}_{\mu_2}(t)$ are statistically independent, i.e., $f_{1,n} \neq f_{2,m}$ for all $n = 1, 2, \ldots, N_1$ and $m = 1, 2, \ldots, N_2$, then we can express the joint pdf of $\hat{p}_{\mu_1}(t)$ and $\hat{p}_{\mu_2}(t)$, $\hat{p}_{\mu_1 \mu_2}(x_1, x_2)$, as follows:

$$\hat{p}_{\mu_1 \mu_2}(x_1, x_2) = \hat{p}_{\mu_1}(x_1) \cdot \hat{p}_{\mu_2}(x_2) \quad (39)$$

Finally, the transformation of the Cartesian coordinates $(x_1, x_2)$ to polar coordinates $(z, \theta)$ by using

$$x_1 = z \cos \theta \quad (40a)$$
and

\[ x_2 = z \sin \theta \]  

allows us to derive the joint pdf of the amplitude \( \hat{x}(t) \) and phase \( \hat{\phi}(t) \), in the following way:

\[
\begin{align*}
\hat{p}_x(z, \theta) &= z \hat{p}_{\mu_1 \mu_2} (z \cos \theta, z \sin \theta) \\
\hat{p}_\phi(z, \theta) &= z \hat{p}_{\mu_1} (z \cos \theta) \cdot \hat{p}_{\mu_2} (z \sin \theta) \\
\hat{p}_{\phi}(z, \theta) &= z \hat{p}_{\mu_1} (z \cos \theta - \rho \cos \theta) \\
&\quad \cdot \hat{p}_{\mu_2} (z \sin \theta - \rho \sin \theta).
\end{align*}
\]

From (41c), we can directly compute the desired amplitude pdf \( \hat{p}_x(z) \) and phase pdf \( \hat{p}_\phi(\theta) \) of the deterministic simulation system according to

\[
\begin{align*}
\hat{p}_x(z) &= \int_0^\pi \hat{p}_{\mu_1} (z \cos \theta - \rho \cos \theta) \\
&\quad \cdot \hat{p}_{\mu_2} (z \sin \theta - \rho \sin \theta) d\theta \\
\hat{p}_\phi(\theta) &= \int_0^\infty \hat{p}_{\mu_1} (z \cos \theta - \rho \cos \theta) \\
&\quad \cdot \hat{p}_{\mu_2} (z \sin \theta - \rho \sin \theta) dz.
\end{align*}
\]

The result of the evaluation of (42a) and (42b) by using (32) is shown in the Fig. 6(a) and (b), respectively. Thereby, the Doppler coefficients \( c_{\mu_1 \mu_2} \) are computed from (37), and the number of sinusoids \( N \) has been restricted to \( N = 7 \). For the reasons of comparison, we present also in the same figures the behavior of the ideal pdf's \( p_x(z) \) and \( p_\phi(\theta) \) according to (9) and (10), respectively.

The proposed method not only applies to the approximation of Gaussian processes or stochastic processes derived from Gaussian processes (e.g., Rayleigh processes, Rice processes, and lognormal processes), but can also be used for the approximation of other kinds of distribution functions such as the Nakagami distribution.

The Nakagami distribution [19], also denoted as the \( m \) distribution, offers a greater flexibility and often tends to fit the pdf of the received signal amplitude computed from experimental data of certain urban channels better than the common used Rayleigh or Rice distribution [1]. The Nakagami distribution with parameters \( \Omega \) and \( m \) is defined by

\[
\begin{align*}
p_x(z) &= \frac{2m^m z^{2m-1} e^{-m(z/\Omega)^2}}{\Gamma(m)\Omega^m}, \quad m \geq 1/2, \quad z \geq 0
\end{align*}
\]

where \( \Gamma(\cdot) \) is the Gamma function, \( \Omega = \langle \xi(t)^2 \rangle \) denotes the time average power of the received signal strength, and \( m = \langle \xi(t)^2 \rangle^2 / \langle \xi(t)^2 - \langle \xi(t)^2 \rangle^2 \rangle^2 \) is the inverse of the normalized variance of \( \xi(t) \).

We remark that for \( m = 1/2 \) and \( m = 1 \), the Nakagami distribution becomes a one-sided Gaussian and Rayleigh distribution, respectively. Finally, the Rice and lognormal distribution also can be approximated by the Nakagami distribution [19], [20]. For further details on the derivation and simulation of Nakagami fading channel models, we refer to [21] and [22].

In order to find a suitable set for the Doppler coefficients \( \{ c_{\mu_1 \mu_2} \} \) so that the amplitude pdf of the proposed simulation system \( \hat{p}_x(z) \) fits the Nakagami distribution (43), we proceed in a similar way as we have successfully done for the minimization of the \( L_2 \) norm (36) in connection with Gaussian distributions. We only have to replace in (36) the Gaussian distribution \( p_{\mu_i}(x) \) by the Nakagami distribution \( p_x(z) \) as...
defined by (43), and \( \tilde{P}_m(x) \) has to be replaced by \( \tilde{P}_m(x) \) as introduced by (42a). The optimization results for the Nakagami distribution are shown for various values of \( m \) in Fig. 7, whereby in each case \( N_1 = N_2 = 10 \) sinusoids have been used.

Weibull distributions can be derived from uniform distributions, and, on the other hand, uniform distributions can be derived from Gaussian distributions. Consequently, the derivation of deterministic Weibull processes is straightforward and will not be investigated in detail.

B. Determination of Optimal Discrete Doppler Frequencies

In the previous section, we discussed a method, which enables us to compute the Doppler coefficients \( c_{\mu,n} \) in such a way that the pdf \( P_{\tilde{\mu}}(x) \) of the deterministic process \( \tilde{\mu}(t) \) results in an optimal approximation of the Gaussian pdf \( P_{\mu}(x) \) of the stochastic process \( \mu(t) \). In the present section, we will see that the discrete Doppler frequencies \( f_{\mu,n} \) can be determined in such a way that the acf \( \gamma_{\mu,\mu}(t) \) of the deterministic process \( \tilde{\mu}(t) \) results in an optimal approximation of any desired acf \( \gamma_{\mu,\mu}(t) \) of the stochastic process \( \mu(t) \) within an appropriate time interval.

The method discussed here is a modification of the \( L_p \)-norm method [10]. The proposed (modified) \( L_p \)-norm method assumes that the Doppler coefficients \( c_{\mu,n} \) are fixed and given by (37), whereas optimal values for the discrete Doppler frequencies \( f_{\mu,n} \) can be found numerically by minimizing the \( L_p \)-norm for \( p = 2 \), i.e., the square root of the mean-square-error norm

\[
E_{\mu,\mu}^{(2)} := \left[ \frac{1}{T_0} \int_0^{T_0} \left( \gamma_{\mu,\mu}(t) - \gamma_{\mu,\mu}(t) \right)^2 dt \right]^{1/2}
\]

where \( \gamma_{\mu,\mu}(t) \) is given by (24a), \( \gamma_{\mu,\mu}(t) \) can be any desired acf (e.g., resulting from “snapshot” measurements of real-world mobile channels), and \( T_0 \) is defined below and denotes an appropriate time interval \([0, T_0]\) over which the approximation of \( \gamma_{\mu,\mu}(t) \) is of interest. Our investigations of the performance of the method presented above will be restricted to both Doppler power spectral densities \( S_{\mu}(f) \) as introduced in Section II (i.e., Jakes and Gaussian).

For the Jakes psd and the frequency-nonshifted Gaussian psd, we obtain—by using (6) and (8)—for the acf \( \gamma_{\mu,\mu}(t) \) the expression

\[
\gamma_{\mu,\mu}(t) = \frac{\sigma_0^2 f_0 (2\pi f_{\text{max}})^2}{\sigma_0^2 (\pi f_{\text{max}})^2} \quad \text{(Jakes)}
\]

\[
\gamma_{\mu,\mu}(t) = \frac{e^{-\frac{\pi f_{\text{max}}^2}{\sigma_0^2}}}{\sqrt{2\pi}} \quad \text{(Gauss)}
\]

Numerical investigations have shown that for these acf’s, appropriate values for \( T_0 \), i.e., the upper bound of the integral in (44), are given if \( T_0 \) is adapted to the number of sinusoids \( N_t \) according to

\[
T_0 = \left\{ \begin{array}{ll}
\frac{N_t}{2f_{\text{max}}} & \quad \text{(Jakes)} \\
\frac{N_t}{\sqrt{f_0^2 / 2}} & \quad \text{(Gauss)}
\end{array} \right.
\]

The evaluation of the error function \( E_{\mu,\mu}^{(2)} \) [see (44)] with the resulting optimized discrete Doppler frequencies \( f_{\mu,n} = f_{\mu,n}^{(\text{opt})} \) is shown for the Jakes psd and the Gaussian psd in the Fig. 8(a) and (b), respectively. For the sake of comparison, we also have shown in these figures the results obtained by applying two other efficient parameter computation techniques, which will be concisely described below.

1) Method of Equal Areas [5], [6]: The principle of this method is as follows. Select a set of discrete Doppler frequencies \( \{f_{\mu,n}\} \) such that in a manner that in the range of \( f_{\mu,n+1} \leq f < f_{\mu,n} \), the area under the (symmetrical) Doppler psd \( S_{\mu,\mu}(f) \) is equal to \( \sigma_0^2/(2N_t) \) for all \( n = 1, 2, \ldots, N_t \) with \( f_{\mu,0} = 0 \). The application of the method of equal areas to the Jakes psd \( S_{\mu,\mu}(f) = S_{\mu,\mu}(f) / 2 \) [see (5)] gives us for the discrete Doppler frequencies

\[
f_{\mu,n} = f_{\text{max}} \sin \left( \frac{\pi n}{2N_t} \right)
\]

for all \( n = 1, 2, \ldots, N_t \) and \( i = 1, 2 \).

Whereas for the frequency-nonshifted Gaussian psd [see (7) with \( f_0 = 0 \)], no closed-form expression for the discrete
Doppler frequencies $f_{i,n}$ exists, and we must calculate the discrete Doppler frequencies $f_{i,n}$ by finding the zeros of

$$\frac{n}{N_i} - \text{ctf} \left( \frac{f_{i,n}}{f_c} \sqrt{n^2 + 2} \right) = 0 \quad (48)$$

for all $n = 1, 2, \ldots, N_i$ and $i = 1, 2$.

The Doppler coefficients are exactly the same as introduced by (37). For a comprehensive derivation and discussion of the method of equal areas, we refer to [6], where also some applications and simulation results can be found.

2) Method of Exact Doppler Spread: This procedure is due to its simplicity and high performance predestinated to the approximation of the acf $\hat{r}_{\mu,\nu}(t)$ as given by (45a), i.e., when the Jakes psd is of interest. The method can easily be derived by using the integral representation [23]

$$J_0(z) = \frac{2}{\pi} \int_0^{\pi/2} \cos(z \sin \alpha) \, d\alpha \quad (49)$$

and replacing the above integral by a series expansion as follows:

$$J_0(z) = \lim_{N_i \to \infty} \frac{2}{\pi} \sum_{n=1}^{N_i} \cos(z \sin \alpha_n) \Delta \alpha \quad (50)$$

where $\alpha_n := \pi(2n - 1)/(4N_i)$ and $\Delta \alpha := \pi/(2N_i)$. Hence, we can write by using (45a) and (50)

$$r_{\mu,\nu}(t) \approx \frac{\sigma_0^2}{N_i} J_0(2\pi f_{\text{max}} t) = \lim_{N_i \to \infty} \frac{\sigma_0^2}{N_i} \sum_{n=1}^{N_i} \cos(2\pi f_{i,n} t) \quad (51)$$

where

$$f_{i,n} = f_{\text{max}} \sin \left[ \frac{\pi}{2N_i} (n - 1/2) \right] \quad (52)$$

Finally, as for finite values of $N_i$, we require $r_{\mu,\nu}(t) \approx \hat{r}_{\mu,\nu}(t)$, and, thus, the comparison of (51) with (24a) leads to

$$\hat{r}_{\mu,\nu}(t) \approx \frac{\sigma_0^2}{N_i} \sum_{n=1}^{N_i} \cos(2\pi f_{i,n} t) \quad (53)$$

A comparison of (53) with (24a) shows that the discrete Doppler frequencies $f_{i,n}$ are identical with (52), and the Doppler coefficients $c_{i,n}$ can immediately be identified with those given by (37).

The Doppler psd $S_{\mu,\nu}(f)$ is often characterized by the Doppler spread $B_{\mu,\nu}$, that is, the square root of the second central moment (variance) of $S_{\mu,\nu}(f)$. In [6], it has been shown that the Doppler spread of deterministic simulation models $\hat{B}_{\mu,\nu}$ depends directly on the discrete Doppler frequencies $f_{i,n}$ and Doppler coefficients $c_{i,n}$. Consequently, the approximation quality $B_{\mu,\nu} \approx \hat{B}_{\mu,\nu}$ is mainly affected by the employed parameter computation method. In Section IV, we will see that for the Jakes psd, the proposed method always results in a Doppler spread of the simulation model $\hat{B}_{\mu,\nu}$ that is identical with the Doppler spread of the analytical model $B_{\mu,\nu}$, i.e., $\hat{B}_{\mu,\nu} = \hat{B}_{\mu,\nu}$. Therefore, we have designated the procedure as “method of exact Doppler spread.”

Clearly, the method of exact Doppler spread is adapted to the Jakes psd, but a comparison of (47) and (52) motivates us— as a heuristic approach—to replace in (48) $n$ by $n - 1/2$. This enables the computation of the discrete Doppler frequencies $f_{i,n}$ for the frequency nonshifted Gaussian psd by finding the zeros of

$$\frac{2n - 1}{2N_i} - \text{ctf} \left( \frac{f_{i,n}}{f_c} \sqrt{n^2 + 2} \right) = 0 \quad (54)$$

for all $n = 1, 2, \ldots, N_i$ and $i = 1, 2$. The Doppler coefficients $c_{i,n}$ remain unchanged and are further given by (37).

After the calculation of the discrete Doppler frequencies $f_{i,n}$ and the Doppler coefficients $c_{i,n}$, by applying one of the three methods described above, the acf of the simulation model $\hat{r}_{\mu,\nu}(t)$ can be computed by means of (24a). The result for $r_{\mu,\nu}(t)$ that has been obtained by applying the $L_2$-norm method, the method of equal areas, and the method of exact Doppler spread are presented for the Jakes psd in Fig. 9(a) and for the frequency-nonshifted Gaussian psd in Fig. 9(b). In all cases, the number of sinusoids $N_i$ was determined by $N_i = 7$. Observe that we have also depicted in Fig. 9(a) and (b) the corresponding acf’s of the analytical model $\hat{r}_{\mu,\nu}(t) = \sigma_0^2 \delta(t) f_{\text{max}} (2\pi f_{\text{max}} t)$ and $\hat{r}_{\mu,\nu}(t) = \sigma_0^2 \exp[-(f_{\text{max}} t)^2/(2\ln 2)]$. This allows us directly to compare and criticize the performance of the various parameter computation methods. The $L_2$-norm method is more complex and requires higher numerical effort than the method of exact Doppler spread, but surprisingly, we realize from Figs. 8(a) and 9(a) that these two methods are yielding nearly equivalent and excellent approximation results in the interval $0 \leq t \leq T_0$.

This, due to its simplicity and high-approximation quality, we prefer for the Jakes psd the method of exact Doppler spread. We gather from the Figs. 8(b) and 9(b) that for the Gaussian psd, the method of exact Doppler spread results in essential smaller approximation errors than the method of equal areas. A comparison of the performance of these two methods with the $L_2$-norm method reveals undoubtedly the advantage of that optimization procedure. Therefore, we prefer for the Gaussian psd the $L_2$-norm method.

Finally, we mention that the period of the deterministic process $\hat{\mu}(t)$ see (22) is inversely proportional to the greatest common divisor of the discrete Doppler frequencies $f_{i,n}$. For all discussed methods, the greatest common divisor can be made small enough for realistic values of $N_i$, say $N_i \geq 7$, simply by increasing the word length of the discrete Doppler frequencies $f_{i,n}$, and thus, the period of $\hat{\mu}(t)$ can be made extremely large.

C. Determination of the Doppler Phases

The Doppler phases $\theta_{i,n}$ are uniformly distributed random variables, which can be obtained simply by means of a random generator with uniform distribution over the interval $(0, 2\pi]$. But for deterministic simulation models, where the Doppler coefficients $c_{i,n}$ and the discrete Doppler frequencies $f_{i,n}$ are all determined by using deterministic methods, it is an obvious idea to introduce also for the computation of the Doppler phases $\theta_{i,n}$ a deterministic approach.
Therefore, we consider the following standard phase vector \( \hat{\psi}_i \) with \( N_i \) deterministic phase components:

\[
\hat{\psi}_i = \left( 2\pi \frac{1}{N_i+1}, 2\pi \frac{2}{N_i+1}, \ldots, 2\pi \frac{N_i}{N_i+1} \right)
\]

and we group the Doppler phases \( \theta_{\rho_n} \) of the \( N_i \) sinusoids to a so-called Doppler phase vector \( \hat{\theta}_i \) in accordance with

\[
\hat{\theta}_i = (\theta_{\rho_1}, \theta_{\rho_2}, \ldots, \theta_{\rho_{N_i}})
\]

for \( i = 1, 2 \). Now, the components of the Doppler phase vector \( \hat{\theta}_i \) can be identified with the components of the standard phase vector \( \hat{\psi}_i \) after permuting the components of \( \hat{\psi}_i \). In this way, for a given number of sinusoids \( N_i \), one can construct \( N_i! \) different Doppler phase vectors \( \hat{\theta}_i \) with equally distributed components.

Let us assume that the real deterministic processes \( \tilde{\mu}_1(t) \) and \( \tilde{\mu}_2(t) \) are uncorrelated, then it can be shown that the corresponding Doppler phases \( \theta_{\rho_{n1}} \) and \( \theta_{\rho_{n2}} \) have no influence on the statistical properties of the complex deterministic processes \( \tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t) \). Consequently, for one and the same sets of Doppler coefficients \( \{c_{\xi n}\} \) and discrete Doppler frequencies \( \{f_{\xi n}\} \), it is possible to construct \( N_i! \) different sets of Doppler phases \( \{\theta_{\rho_{n1}}\} \), and, thus, \( N_i! \cdot N_i! \) different complex deterministic processes \( \tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t) \) with different time behavior, but identical statistics can be realized. This is of great advantage for simulating channels for mobile digital transmission systems with frequency hopping. In such cases, a frequency hop necessitates a new permutation of the Doppler phases.

On the other side, if \( \tilde{\mu}_1(t) \) and \( \tilde{\mu}_2(t) \) are correlated, then the Doppler phases \( \theta_{\rho_{n1}} \) have an influence on the cross-correlation function \( \hat{\nu}_{\mu_1\mu_2}(t) \), as can easily be seen by considering (25). In this case, the Doppler phases \( \theta_{\rho_{n1}} \) can be designed such that they control the higher order statistics of \( \tilde{\xi}(t) = [\tilde{\mu}(t) + m(t)] \), e.g., the simulation model’s lcr and the adf’s [24].

IV. HIGHER ORDER STATISTICAL PROPERTIES OF DETERMINISTIC SIMULATION MODELS

In this section, we derive for the deterministic simulation models of Fig. 3(a) and (b) analytical expressions for the lcr \( \tilde{N}_\xi(r) \) and the adf’s \( \tilde{T}_\xi(r) \). The analytical expressions do not only allow us to avoid the determination of \( \tilde{N}_\xi(r) \) and \( \tilde{T}_\xi(r) \) from time-consuming simulations of the channel output signal \( \tilde{\xi}(t) \), but also enable us to study the degradation effects due to the selected parameter computation method and the limited number of sinusoids \( N_i \). Moreover, we investigate the pdf of fading time intervals by using the Jakes and Gaussian psd’s.

In Section III-A, it has been shown that the pdf of the deterministic process \( \tilde{\mu}_1(t) \), \( \tilde{\mu}_2(t) \), is nearly identical with the pdf of the stochastic process \( \mu_1(t) \), \( \mu_2(t) \), if the number of sinusoids \( N_i \) is sufficiently large, say, \( N_i \geq 7 \). On the assumption that \( \tilde{\mu}_1(x) = \mu_1(x) \) yields, the lcr of the deterministic simulation model \( \tilde{N}_\xi(r) \) is furthermore defined by (12). One merely has to replace the quantities of the analytical model \( \alpha \) and \( \beta \) [consider, therefore, (13) and (14)] by the corresponding quantities of the simulation model \( \hat{\alpha} \) and \( \hat{\beta} \). Thus

\[
\hat{\alpha} = \left( 2\pi f_\rho - \frac{\bar{\xi}_0}{\psi_0} \right) / \sqrt{2\hat{\beta}}
\]

\[
\hat{\beta} = -\psi_0 - \frac{\bar{\psi}_0^2}{\psi_0}
\]

where \( \bar{\psi}_0, \bar{\xi}_0 \) are related by the acf \( \hat{\nu}_{\mu_1\mu_1}(t) \) [see (24a)] and ccf \( \hat{\nu}_{\mu_1\mu_2}(t) \) [see (25)] of the simulation model as follows:

\[
\dot{\bar{\psi}}_0 = \frac{d}{dt} \hat{\nu}_{\mu_1\mu_1}(t) \bigg|_{t=0} = \tilde{\nu}_{\nu_{\mu_1\mu_1}}(0)
\]

\[
\dot{\bar{\xi}}_0 = \frac{d}{dt} \hat{\nu}_{\mu_1\mu_2}(t) \bigg|_{t=0} = \tilde{\nu}_{\nu_{\mu_1\mu_2}}(0)
\]

and \( \bar{\psi}_0 = \hat{\nu}_{\mu_1\mu_1}(0) = \sum_{n=1}^{N_i} \frac{\bar{\xi}_n}{\sum_{n=1}^{N_i} \bar{\xi}_n} / 2 \). Note that if the Doppler coefficients \( c_{\xi n} \) are as given by (37), then \( \bar{\psi}_0 = \psi_0 \) yields.
A. Higher Order Statistical Properties by Using the Jakes PSD

The Jakes psd (5) is a symmetrical function, and, thus, it follows from the properties of the Fourier transform that the ccf is zero, i.e., \( \mu_1(t) \) and \( \mu_2(t) \) are uncorrelated. Now, we impose the same requirement on the simulation system. From (25), it follows that \( \tilde{r}_{\mu_1\mu_2}(t) = 0 \) can easily be achieved for all design methods discussed here by selecting the number of sinusoids such that \( N_1 = N_2 - 1 \) yields. Then, \( N_1 \) is unequal to \( N_2 \), and per definition the discrete Doppler frequencies are designed in such a way that \( f_{f_n} \neq \pm f_{p,m} \) holds for all \( n = 1, 2, \ldots, N_1 \) and \( m = 1, 2, \ldots, N_2 \). Thus, it follows from (57)–(60) in connection with (24a) that

\[
\tilde{\alpha} = 2\pi f_p / \sqrt{2\tilde{\beta}} \quad \text{(61)}
\]

and

\[
\tilde{\beta} = -\frac{\psi_0}{2\pi} = 2\pi^2 \sum_{n=1}^{N_1} (c_{n,\nu} f_{4,n})^2 \quad \text{(62)}
\]

The evaluation of the above equation by using the method of exact Doppler spread can be performed by substituting (37) and (52) in (62), i.e.,

\[
\tilde{\beta} = (2\pi \sigma_0 f_{\text{max}})^2 \cdot \frac{1}{N_1} \sum_{n=1}^{N_1} \sin^2 \left[ \frac{\pi}{2N_1} \left( n - \frac{1}{2} \right) \right] \quad \text{(63)}
\]

By taking notice of \( \psi_0 = \sigma_0^2 \) and making use of the relation

\[
\frac{1}{N_1} \sum_{n=1}^{N_1} \sin^2 \left[ \frac{\pi}{2N_1} \left( n - \frac{1}{2} \right) \right] = \frac{1}{2} \quad \text{(64)}
\]

we finally obtain for \( \tilde{\alpha} \) and \( \tilde{\beta} \) the result

\[
\tilde{\alpha} = \frac{1}{\sqrt{\psi_0}} \frac{f_p}{f_{\text{max}}} 
\]

\[
\tilde{\beta} = 2\psi_0 (\pi f_{\text{max}})^2 \quad \text{(66)}
\]

A comparison of (20a) and (21a) with (65) and (66) reveals that \( \tilde{\alpha} = \alpha \) and \( \tilde{\beta} = \beta \), i.e., the approximation error is zero, and the Doppler spreads \( B_{\tilde{\mu}_1} \) and \( B_{\tilde{\mu}_2} \) are identical too because

\[
B_{\tilde{\mu}_1} = \sqrt{\beta / \psi_0 / (2\pi)} \quad \text{and} \quad B_{\tilde{\mu}_2} = \sqrt{\beta / \psi_0 / (2\pi)}. \]

In a similar way, we can compute \( \beta \) by using the method of equal areas. In this case, we obtain \( \beta = \beta (1 + 1/N_2) \) and \( \tilde{\beta} \) tends to \( \beta \) when \( N_2 \rightarrow \infty \). Let us for a moment assume that the Doppler frequency of the direct component \( f_p \) is equal to zero, i.e., \( f_p = 0 \), then \( \alpha = \tilde{\alpha} = 0 \) and the lcr \( \tilde{N}_\xi(r) \) reduces to (17). Now, the relative error of \( \tilde{N}_\xi(r) \) can be expressed by

\[
\varepsilon_{\tilde{N}_\xi} = \frac{\tilde{N}_\xi(r) - N_\xi(r)}{N_\xi(r)} = \frac{\sqrt{\beta} - \sqrt{\beta}}{\sqrt{\beta}}. \quad \text{(67)}
\]

The substitution of \( \beta = \beta (1 + 1/N_2) \) in (67) and the use of the approximation \( \sqrt{1 + \frac{1}{N_2}} \approx 1 + \frac{1}{2N_2} \) allows us to express the relative error \( \varepsilon_{\tilde{N}_\xi} \) directly as function of the number of sinusoids

\[
\varepsilon_{\tilde{N}_\xi} \approx \frac{1}{2N_2}. \quad \text{(68)}
\]

The preceding equation shows that by using the method of equal areas, about 25 sinusoids are required for the design of \( \tilde{\mu}_1(t) \) and \( \tilde{\mu}_2(t) \) in order to reduce the relative error of \( \tilde{N}_\xi(r) \) to 2%.

It should be noted that for the \( L_2 \)-norm method, no closed-form expression for \( \tilde{\beta} \) can be derived. The use of that method leads after the numerical evaluation of (62) to the results for the normalized quantity \( \beta / f_{\text{max}}^2 \) as depicted in Fig. 10(a). This figure also shows the behavior of \( \beta / f_{\text{max}}^2 \) as function of the number of sinusoids \( N_2 \) by using the method of equal areas and the method of exact Doppler spread.

Next, we consider the deterministic simulation model of Fig. 3(a), and we calculate directly the lcr \( \tilde{N}_\xi(r) \) by counting the number of crossings per second at which the simulated channel output signal \( \tilde{\xi}(t) \) crosses a specified signal level \( r \) with positive slope. Therefore, we have chosen a maximum Doppler frequency \( f_{\text{max}} \) [see (5)] of \( f_{\text{max}} = 91.73 \text{ Hz} \) that
corresponds to a mobile speed of 110 km/h and a carrier frequency of 900 MHz. The variance $\sigma_d^2$ was normalized to $\sigma_0^2 = 1/2$. The parameters of the deterministic simulation system have been designed according to the method of exact Doppler spread with $N_1 = 7$ and $N_2 = 8$ sinusoids so that the cross correlation between the real part $j(t)$ and the imaginary part $\tilde{j}(t)$ is zero. For the realization of the discrete-time deterministic simulation system, we have used a sampling interval $T$ of $T = 3 \cdot 10^{-4}$ s, and altogether $K = 4 \cdot 10^5$ samples of the deterministic process $\tilde{f}(K)$, $k = 0, 1, \ldots, K - 1$, have been simulated for the evaluation of the lcr $N_\text{lc}(r)$. The result for the normalized lcr of the deterministic simulation model $N_\text{lc}(r)/f_{\text{max}}$ is presented in Fig. 11(a). This figure demonstrates for a deterministic Rice process with $\rho = 1$ the influence of the Doppler frequency of the direct component $f_d$ on the behavior of $N_\text{lc}(r)$. For the reasons of comparison, we also have shown in Fig. 11(a) the lcr of the (ideal) analytical model $N_\text{lc}(r)$ as defined by (12). It can be seen that the lcr of the simulation model $N_\text{lc}(r)$ coincides extremely closely with the lcr of the theoretical model $N_\text{lc}(r)$ by applying the method of exact Doppler spread.

**(B. Higher Order Statistical Properties by Using the Gaussian PSD)**

The Gaussian psd (7) is for $f_d \neq 0$, an unsymmetrical function. Consequently, the ccf $r_{\mu_1,\mu_2}(t)$ is unequal to zero, and, thus, $\mu_1(t)$ and $\mu_2(t)$ are correlated. Let the stochastic processes $\mu_1(t)$ and $\mu_2(t)$ [see Fig. 2(b)] be uncorrelated, then we yield after some simple computations the following expressions for the acf $r_{\mu_1,\mu_1}(t)$ and ccf $r_{\mu_1,\mu_2}(t)$:

$$
\begin{align*}
    r_{\mu_1,\mu_1}(t) &= \frac{1}{2}[r_{\nu_1,\nu_1}(t) + r_{\nu_2,\nu_2}(t)] \cos(2\pi f_s t) \\
    r_{\mu_1,\mu_2}(t) &= \frac{1}{2}[r_{\nu_1,\nu_1}(t) + r_{\nu_2,\nu_2}(t)] \sin(2\pi f_s t)
\end{align*}
$$

resulting for the frequency-shifted Gaussian psd in

$$
\begin{align*}
    \psi_0 &= -\psi_0 \left[ 2(\pi f_c / \sqrt{2})^2 + (2\pi f_s)^2 \right] \\
    \phi_0 &= \psi_0 \cdot 2\pi f_s
\end{align*}
$$

respectively.

In a similar way, the corresponding quantities of the deterministic simulation model can be derived. The result is

$$
\begin{align*}
    \psi_0 &= -\pi^2 \left[ \sum_{n=1}^{N_1} (c_{1,n} f_{1,n})^2 + \sum_{n=1}^{N_2} (c_{2,n} f_{2,n})^2 \right] - \psi_0 (2\pi f_s)^2 \\
    \phi_0 &= \psi_0 \cdot 2\pi f_s
\end{align*}
$$

where we have used $\hat{\nu}_d(t) = \psi_0 = \psi_0$, i.e., the Doppler coefficients are given by $c_{1,n} = \sigma_0 \sqrt{2/N_1}$ and $c_{2,n} = \sigma_0 \sqrt{2/N_2}$. A comparison of (70b) with (71b) shows that $\hat{\beta} = \hat{\alpha}$. Now, we are able to compute the quantities $\hat{\alpha}$ and $\hat{\beta}$ which determine the approximation quality of the deterministic simulation model’s lcr. Therefore, we substitute (71a) and (71b) in (57) and (58) and obtain finally

$$
\begin{align*}
    \hat{\alpha} &= \pi \sqrt{\frac{2}{\beta}} (f_d - f_s) \\
    \hat{\beta} &= \pi^2 \left[ \sum_{n=1}^{N_1} (c_{1,n} f_{1,n})^2 + \sum_{n=1}^{N_2} (c_{2,n} f_{2,n})^2 \right]
\end{align*}
$$

Equation (72) allows further observations. When $\hat{\beta} \rightarrow \beta$, $\hat{\alpha}$ tends to $\alpha$, and, consequently, $N_\text{lc}(r)$ tends to $N_\text{lc}(r)$. Obviously, the quantity $\hat{\beta}$ is responsible for the approximation quality of the higher order statistical properties of the deterministic simulation model. For all parameter computation methods discussed in Section III, $\hat{\beta}$ cannot be expressed in a closed-form for the Gaussian psd so that (73) has to be evaluated numerically. Fig. 10(b) shows the numerical evaluation results for the normalized quantity $\hat{\beta}/f_s^2$, which have been obtained by employing all parameter computation methods investigated in the present paper.
Fig. 12. The pdf of the fading time intervals $p_{\text{f}}(\tau, r)$ of the deterministic Rayleigh process $\xi(t)$ by using the Jakes psd ($\sigma_0^2 = 1/2, f_{\text{max}} = 91$ Hz, and $\rho = 0$) if the signal level $\xi(t) = r$ is (a) $r = 0.1$ and (b) $r = 1$ as well as by using the Gaussian psd ($\sigma_0^2 = 1/2, f_c = f_{\text{max}}\sqrt{\ln 2}$, and $\rho = 0$) if (c) $r = 0.1$ and (d) $r = 1$.

Now, let us consider Fig. 3(b) and let us determine for the frequency-shifted Gaussian psd the lcr $\bar{N}_\xi(\tau)$ from the simulated sequence $\hat{\xi}(kT)$, $k = 0, 1, \ldots, K = 1$, in the same way as we have already done for the Jakes psd. Therefore, we have selected the following model parameters: $T = 3\cdot 10^{-4}$ s, $K = 4\cdot 10^5$, $f_c = \sqrt{\ln 2} \cdot f_{\text{max}}$, $f_{\text{max}} = 91.73$ Hz, $\rho = 1$, and $f_\rho = 0$, and in order to study the influence of the frequency shift $f_c$, we have used the values $f_c \in \{0, \frac{2}{3} f_{\text{max}}, f_{\text{max}}\}$.

Observe by considering (21) that the 3-dB cutoff frequency $f_c = \sqrt{\ln 2} \cdot f_{\text{max}}$ was selected such that the quantity $\beta$ is identical for both power spectral densities (Jakes and Gaussian). The discrete Doppler frequencies $f_{\text{D}, \nu}$ and Doppler coefficients $c_{\nu, \tau}$ have been determined by the method of exact Doppler spread with $N_1 = 25$ and $N_2 = 26$. The results for the normalized lcr’s of the deterministic simulation model $\bar{N}_\xi(\tau)/f_{\text{max}}$ as well as of the analytical model $N_\xi(\tau)/f_{\text{max}}$ are shown in Fig. 11(b). A comparison of Fig. 11(a) with (b) shows the same result for both the analytical and the simulation model: identical lcr’s are observed—as it was expected by the theory—although the underlying Doppler power spectral densities are quite different.

In a similar way, an expression for the adf’s of the simulation model $\bar{F}_\xi(\tau)$ can be derived. One merely has to replace in (18) $\bar{N}_\xi(\tau)$ by $N_\xi(\tau)$ and $\bar{P}_\xi(\tau)$ by $\bar{P}_\xi(\tau)$. Thus, the analytical and numerical investigations of $\bar{F}_\xi(\tau)$ are straightforward and will not be presented here.

C. Distribution of the Fading Time Intervals

The performance of mobile communication systems can mainly be improved by an efficient design of the interleaver/deinterleaver and the channel coding/decoding unit. For the specification of these units, not only the knowledge of the lcr and adf’s is of special interest, but also the distribution of the fading time intervals. The rest of this section is focused on the pdf of the fading time intervals that will be denoted here by $p_0(\tau, r)$. This pdf $p_0(\tau, r)$ is the conditional probability density that the stochastic process $\xi(t)$ crosses a certain level $r$ for the first time at $t_2 = t_1 + \tau$ with positive slope given a crossing with negative slope at time $t_1$. An exact analytical evaluation of $p_0(\tau, r)$ is still an unsolved problem, even for Rayleigh and Rice processes. So, we will determine $p_0(\tau, r)$ experimentally by employing determin-
istic simulation systems. For that purpose, we simulate \( \hat{\xi}(kT) \) with a sampling interval of \( T = 0.5 \times 10^{-4} \text{s} \), and we fix \( \rho = 0 \), i.e., \( \hat{\xi}(kT) \) is a deterministic Rayleigh process. Typical measurement results of \( \hat{\rho}_0(\tau, r) \) for \( r = 0.1 \) and \( r = 1 \) by evaluating \( 2 \times 10^4 \) fading time intervals \( \tau \) are presented in Fig. 12(a) and (b) for the Jakes psd and in Fig. 12(c) and (d) for the Gaussian psd, respectively, whereby we have applied the method of exact Doppler spread by using various values for the number of sinusoids \( N_c \). The remaining model parameters were \( f_{\text{max}} = 91 \text{ Hz}, \ f_c = f_{\text{max}} \sqrt{\ln 2} \sigma^2_0 = 1/2, \) and \( f_s = 0 \).

Approximative solutions for \( \hat{\rho}_0(\tau, r) \) can be found, for example, in [25]–[29]. The first approximation to \( \hat{\rho}_0(\tau, r) \) is \( \hat{\rho}_1(\tau, r) \), where \( \hat{\rho}_1(\tau, r) \) is the conditional probability density that the stochastic process \( \xi(t) \) passes upwards through a certain level \( r \) at time \( t_2 = t_1 + \tau \) given that \( \xi(t) \) has passed downward through \( r \) at time \( t_1 \). Note, that nothing is said about the behavior of \( \xi(t) \) between \( t = t_1 \) and \( t = t_2 \). In [25], an analytical solution for \( \hat{\rho}_1(\tau, r) \) can be found that tends for deep fades to

\[
p_{1-}(\tau, r) = \frac{d}{dr}\left[ \frac{2}{\pi iT_\xi(r)} I_1 \left( \frac{2}{\pi iT_\xi(r)} e^{-\frac{\pi r^2}{4}} \right) \right], \quad \text{if } r \to 0
\]

(74)

where \( u = \tau/2T_\xi(r) \) and \( I_1(\cdot) \) denotes the modified Bessel function of first order. The graph of this function also is shown for the Rayleigh process \( \xi(t) \) with underlying Jakes and Gaussian psd characteristics in Fig. 12(a) and (c), respectively. These figures impressively show the excellent accordance between the theoretical approximation \( \hat{\rho}_1(\tau, r) \) and the measurement results \( \hat{\rho}_0(\tau, r) \) for deep fades, i.e., for small values of \( r \), where the probability that only short fading time intervals \( \tau \) occur is high. Moreover, Fig. 12 led us to the hypothesis that stochastic processes \( \xi(t) \) with different Doppler psd characteristics, but identical values for the quantities \( \alpha \) and \( \beta \) have identical pdf’s of the fading time intervals \( \hat{\rho}_0(\tau, r) \) for small values of \( r \). Assuming that the hypothesis is true, then some consequences arise for the modeling of real-world mobile channels on the basis of measurements of the statistics of the received signal. We conclude from the above that for the simulation model’s Doppler psd it is not necessary to match exactly the measured Doppler psd, but the approximation quality of the respective acf’s around the time origin is of utmost importance for the modeling of the measured channel statistics.

V. CONCLUSION

In this paper, we have investigated the statistical properties of deterministic simulation models for mobile fading channels. Especially for frequency nonselective deterministic simulation models, we have not only derived analytical expressions for the pdf of the amplitude and phase, but also higher order statistics like lcr and adf’s have been investigated analytically. This makes the time-consuming evaluation and estimation of the statistics from simulated channel output sequences superfluous. Moreover, the derived analytical expressions allow detailed investigations of the degradation effects due to a limited number of sinusoids or due to the applied computation method for the simulation model’s discrete Doppler frequencies and Doppler coefficients, and they provide us with a powerful tool when discussing and comparing the efficiency of different parameter computation methods.

A performance study of three efficient parameter computation methods was also a topic of the present paper, whereby two of the considered methods are new.

In particular, for the often-used Jakes and Gaussian Doppler power spectral densities, it turned out by a comparison of the so-called method of exact Doppler spread with the method of equal areas that the former allows a drastic reduction in the number of sinusoids without producing noteworthy deviations from the desired (ideal) statistics. The third introduced method, which we have named the \( L_2 \)-norm method, requires a greater numerical expenditure than the other two methods. This method develops its full performance by the approximation of measured acf’s obtained from real-world mobile radio channels, where other more conventional parameter computation methods tend to fail.

REFERENCES


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